The Maths behind the Model

This document outlines the simple 'transmission line' model I have used to model the mechanical behaviour of an LP replay cartridge's stylus assembly. It treats the shank as being a mechanical transmission line connecting a mechanical vibration source to a load presented by the suspension and output generator. The model treats the mechanical properties in terms of electronic analogues. e.g. Voltage is used in place of physical displacements, etc. This lets us apply analysis methods normally used for electronics to a mechanical arrangement.

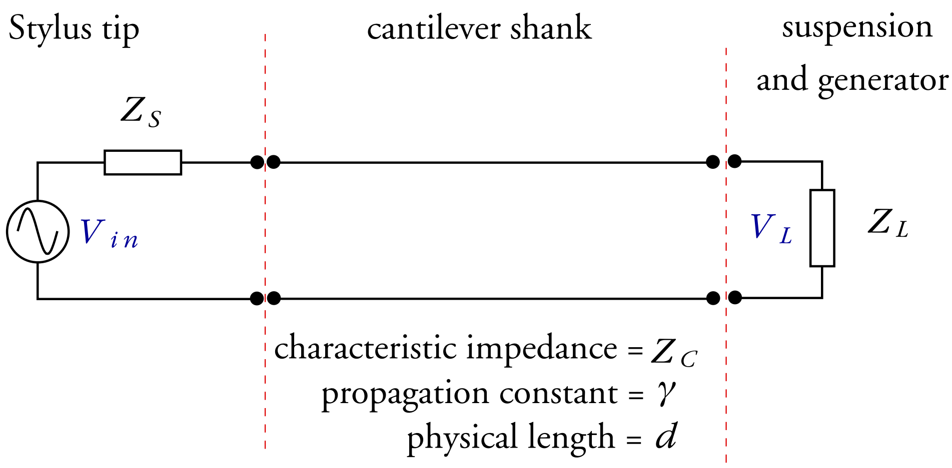


Figure 1 Simple transmission line model of stylus, etc.

In general almost all the values here may be complex. However for an ideal matched system we would have

$$Z_S = Z_L = Z_C$$

where these impedances are all real, and the propagation coefficient would entirely imaginary – i.e. it would have no real part. In this situation the power coupling would be perfect at all frequencies, and there would be no variations in the power/frequency response pattern.

More generally, those conditions will not all be met in most practical cases. This then will tend to give rise to variations in the efficiency of the power coupling and the time/phase delay as a function of frequency.

We can define the signal amplitude coupling ratio (analogous to the voltages) to be

$$A_V = \frac{V_L}{V_{in}}$$

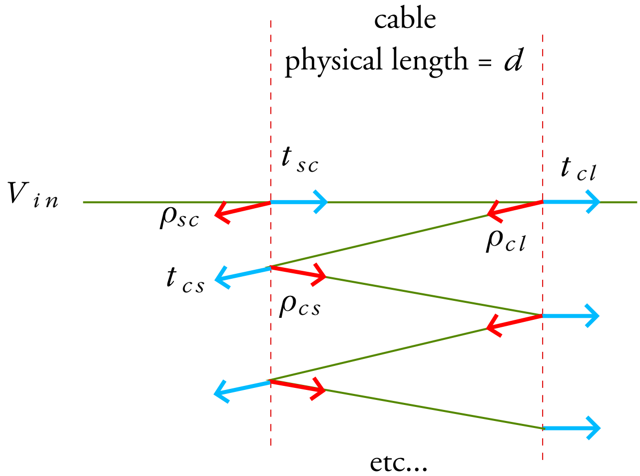
The reflection coefficients at each end of the line will be

$$\rho_{sc} = \frac{Z_C - Z_S}{Z_C + Z_S} \quad ; \quad \rho_{cl} = \frac{Z_L - Z_C}{Z_L + Z_C}$$

The associated transmission coefficients will then be

$$\tau_{sc} = \rho_{sc} + 1 \quad ; \quad \tau_{cl} = \rho_{cl} + 1$$

(N.B. Yes, the positive signs are correct!)



$$V_0 = V_{in}\, t_{sc} \operatorname{Exp}\{-\gamma d\}\, t_{cl}$$

$$V_1 = V_0 \operatorname{Exp}\{-2\gamma d\}\, \rho_{cl}\rho_{cs}$$

$$V_2 = V_0 [\operatorname{Exp}\{-2\gamma d\}\, \rho_{cl}\rho_{cs}]^2$$

$$V_n = V_0 [\operatorname{Exp}\{-2\gamma d\}\, \rho_{cl}\rho_{cs}]^n$$

Figure 2 Cascade of reflected output contributions

We can then analyse the situation when each end of the line may be mismatched causing a sequence of multiple reflections back and forth along the line. This produces an infinite series as shown in Figure 2. The series can be replaced by a standard identity of the form

$$A_V = \frac{T}{1 - B}$$

where

$$T = \tau_{sc}\tau_{cl} \operatorname{Exp}\{-\gamma d\} \qquad ; \qquad B = \rho_{sc}\rho_{cl} \operatorname{Exp}\{-2\gamma d\}$$

and for a loss-free transmission line

$$\gamma = j\frac{2\pi f}{c}$$

where f is the signal frequency, c is the velocity of the signal along the line.

Note, however, that for lossy and/or dispersive transmission lines γ may be complex and include a real term. Also, in practice almost all the factors described here may themselves be frequency dependent and complex! For example, the suspension's (load) impedance is quite likely to be frequency sensitive. Work by Kogan (Shure) has reported deliberately arranging this in order to make cartridges which have a high compliance at low frequency whilst having a lower compliance (and more loss) at high frequency in order to help damp HF resonances.

It should also be noticed that is is common for the mechanical properties of a stylus assembly to differ in the horizontal and vertical planes. In particular, the compliance in the vertical plane tends to be lower than in the horizontal plane. This is because this has to support the playing downforce.

However even using some simple assumptions it becomes possible to model the behaviour of typical styli when playing an LP and get a result that looks similar to the real performance.

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